may not only lead to an incorrect growth rate of disturbances within the linearized problems, but may also distort the evolution amplitude as a function of time t_1 in the nonlinear regime.

Thus, the lack of clarity in presenting the form of the solution, marked by the present referees in commenting on the original version of the study reviewed, must be understood not simply as a wish for an improved style of presentation, but an intrinsically vague formulation of the problem considered. From the revised version of the article it is obvious that the referees' comments have so far not been grasped by the author.

Nevertheless, A. P. Khokhlov's article "The theory of resonance interaction of Tollmien–Schlichting waves" deserves publication in Prikladnaya Mekhika Tekhicheskaya Fizika, though in a form presented following suitable corrections. As follows from the discussion above, however, the problem of whether the results presented in it are asymptotically correct is still open.

EFFECT OF POLYDISPERSION ON SOUND PROPAGATION IN GAS MIXTURES WITH VAPOR AND LIQUID DROPS

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The propagation of nonstationary low-amplitude disturbances in heterogeneous gas mixtures with vapor and liquid drops is one of the current problems of wave dynamics of two-phase systems. Such heterogeneous media are basic working units in energy devices, apparatus of chemical technology, and other devices of contemporary technology. In these cases, to control the flow of various technological processes one often uses calculations and measurements of propagation and absorption rates of acoustic waves. Therefore, investigations of the effects of various physicochemical transformations on the character of disturbance propagation in two-phase gas–drop systems are very valuable.

Despite the number of published studies, the propagation of acoustic waves in vapor–gas–drop systems in the presence of interphase in the presence of mass exchange has so far not been investigated in sufficient detail. Most of the studies in acoustics of gas–vapor–drop media were devoted to the study of propagation of low-intensity waves in multiply disperse systems [1-12]. A number of aspects of the effect of polydispersion on propagation of acoustic disturbances in gas suspensions in the absence of mass exchange was treated earlier in [1, 13]. The problem of sound propagation in polydisperse vapor–gas–drop mixtures has practically not been investigated. In the present study we investigate for the first time the effect of polydispersion on the propagation of low-intensity waves in vapor–gas–drop systems, including effects of nonequilibrium phase transformations.

1. Consider the one-dimensional motion of a polydisperse vapor–gas–drop mixture in an acoustic field, when the disturbances in mixture parameters are small. The basic characteristics of this suspension are the following parameters:

$$n = \int_{a_{\min}}^{a_{\max}} N(a)\,da, \quad \alpha_2 = \int_{a_{\min}}^{a_{\max}} \frac{4}{3}\pi a^3 N(a)\,da, \quad \alpha_2 + \alpha_1 = 1,$$

$$\rho_1 = \alpha_1\rho_1^0, \quad \rho_2 = \alpha_2\rho_2^0 = \int_{a_{\min}}^{a_{\max}} m_2(a)\,N(a)\,da, \quad m_2 = \frac{4}{3}\pi a^3\rho_2^0,$$

$$m = \rho_{20}/\rho_{10}, \; k_j = \rho_{j0}/\rho_{10}, \; j = V,\ G, \; k_V + k_G = 1.$$

Here N(a) is the size distribution function of drops in the suspension with minimum $a_{\min}$ and maximum $a_{\max}$ drop radii, n, α_i, ρ_i^0, ρ_i are the total number of particles per unit volume, the bulk content, and the true and mean densities of the gas phase (i = 1) and of particles (i = 2), m_2 and m are the mass of a single drop and the initial mass content of drops, k_j is the initial concentration of the vapor (j = V) and gas (j = G) components of the gas phase, and

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Kazan'. Tyumen'. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, No. 4, pp. 75-83, July-August, 1993. Original article submitted February 26, 1992.

the superscripts 0 refer to parameters of the initial unperturbed homogeneous state of the mixture.

A system of linear differential equations of motion of the polydisperse vapor–gas–drop mixture is obtained by integrating the linearized equations of motion for a monodisperse suspension [7] over the drop radius a from a_{min} to a_{max}.† In a coordinate system relative to which the unperturbed mixture is at rest the conservation equations of mass and of the momenta of the host phase and of the disperse drops are

$$\frac{\partial\rho_i'}{\partial t}+\rho_{i0}\frac{\partial v_1'}{\partial x}=-\int\limits_{a_{min}}^{a_{max}}\tilde{N}_0\tilde{j}_{V\Sigma}da,$$

$$i=1,V,\ \frac{\partial\tilde{\rho}_2'}{\partial t}+\int\limits_{a_{min}}^{a_{max}}\tilde{N}_0\tilde{m}_{20}\frac{\partial\tilde{v}_2'}{\partial x}da=\int\limits_{a_{min}}^{a_{max}}\tilde{N}_0\tilde{j}_{\Sigma}da, \tag{1.1}$$

$$\rho_{10}\frac{\partial v_1'}{\partial x}+\frac{\partial p_1'}{\partial x}+\int\limits_{a_{min}}^{a_{max}}\tilde{N}_0\tilde{f}\,da=0,\quad \tilde{m}_{20}\frac{\partial\tilde{v}_2'}{\partial t}=\tilde{f},$$

$$\tilde{\rho}_2'=\int\limits_{a_{min}}^{a_{max}}[\tilde{m}_2'\tilde{N}_0+\tilde{m}_{20}\tilde{N}']\,da,\quad \rho_1=\rho_V+\rho_G,\quad p_1=p_V+p_G,$$

where v, p are the velocity and pressure, $j_{V\Sigma}$ is the diffusion flow of the vapor to the drop surface Σ, j_{Σ} is the condensation intensity at the surface of an individual drop, f is the force on an isolated drop due to the host phase, the sign ~ denotes quantities depending on the parameter a, and the prime denotes a parameter perturbation.

The equations of thermal influx to the gas phase, drops, and surfaces of individual drops are written in the form

$$\rho_{10}c_{p1}\frac{\partial T_1'}{\partial t}-\alpha_{10}\frac{\partial p_1'}{\partial t}=-\int\limits_{a_{min}}^{a_{max}}\tilde{N}_0\tilde{q}_{1\Sigma}da,\quad \tilde{m}_{20}c_2\frac{\partial\tilde{T}_2'}{\partial t}=\tilde{q}_{2\Sigma}, \tag{1.2}$$

$$\tilde{q}_{1\Sigma}+\tilde{q}_{2\Sigma}=-\tilde{j}_{\Sigma}l_0,\ \tilde{j}_{V\Sigma}=\tilde{j}_{\Sigma},\ c_{pi}=k_Vc_{pV}+k_Gc_{pG}.$$

Here T is the temperature, c_p is the heat capacity at constant pressure, c_2 is the heat capacity of the incompressible disperse phase, $q_{j\Sigma}$ is the intensity of heat exchange of the j-th phase (j = 1, 2) with the drop surface, and ℓ is the specific heat of vapor formation.

The linearized equations of state of a vapor and gas mixture are represented in the form

$$p_V'=\frac{C_V^2}{\gamma_V\alpha_{10}}(\rho_V'+rk_V\rho_2')+p_{V0}\frac{T_1'}{T_0},$$

$$p_1'=\frac{C_{10}^2}{\gamma_{10}\alpha_{10}}[g_2(\rho_1'+r\rho_2')+g_1(\rho_V'+rk_V\rho_2')]+p_{10}\frac{T_1'}{T_0}, \tag{1.3}$$

$$r=\rho_{10}^0/\rho_{20}^0,\quad b=R_V/R_{10},\quad g_1=(R_V-R_G)/R_{10},\quad g_2=1-k_Vg_1$$

(C is the speed of sound, and γ is the adiabatic index).

For given force, thermal, and massive phase interactions one takes into account the dependencies of the force f, thermal fluxes $q_{j\Sigma}$ (j = 1, 2), and mass exchange intensities $j_{V\Sigma}$ on the oscillation frequency ω [8, 13-15]:

$$\tilde{f}=\tilde{m}_{20}\frac{v_1'-\tilde{v}_2'}{\tilde{\tau}_v^*},\quad \tilde{j}_{V\Sigma}=\tilde{m}_{20}\frac{r}{p_{10}}\frac{p_V'-\tilde{p}_{V\Sigma}'}{\tilde{\tau}_p^*},\quad \tilde{j}_{\Sigma}=m_{20}\frac{r}{p_{10}}\frac{\tilde{p}_{V\Sigma}'-\tilde{p}_{VS}'}{\tilde{\tau}_\beta},$$

$$\tilde{q}_{1\Sigma}=\tilde{m}_{20}\frac{c_{p1}}{m}\frac{T_1'-\tilde{T}_\Sigma'}{\tilde{\tau}_{T1}^*},\quad \tilde{q}_{2\Sigma}=\tilde{m}_{20}c_2\frac{\tilde{T}_2'-\tilde{T}_\Sigma'}{\tilde{\tau}_{T2}^*},\quad \tilde{p}_{VS}'=\frac{l_0\rho_{V0}^0}{(1-rk_V)T_0}\tilde{T}_\Sigma'. \tag{1.4}$$

Here $\tilde{p}_{VS}$ is the partial pressure of saturated vapor, related to the drop surface temperature $\tilde{T}_\Sigma$ by the Clausius–Clapeyron equation, and $\tilde{\tau}^*$ is a complex time, characterizing the dynamics

†An integration procedure for the special case of a single-component mixture of a vapor with drops was treated earlier in [13].

and heat and mass exchange of a single drop with the surrounding gas in a high-frequency acoustic field [8, 15]:

$$\tilde{\tau}_v^* = \tilde{\tau}_v\left[1 + \frac{1-i}{\sqrt{2}}(\omega\tilde{\tau}_{\mu 1})^{1/2}\right]^{-1}, \quad \tilde{\tau}_p^* = \frac{1}{3}\frac{R_V}{R_{10}}(1-k_V)\tilde{\tau}_d\tilde{\varphi}(\tilde{y}),$$

$$\tilde{\tau}_\beta = \frac{1}{3}\sqrt{\frac{2\pi}{\gamma_V}\frac{\gamma_1 C_V a}{\beta C_1^2}}, \quad \tilde{\tau}_{T1}^* = \frac{1}{3}\frac{\alpha_{10}}{\alpha_{20}}\tilde{\tau}_{\lambda 1}\eta_1(\tilde{z}_1), \quad \tilde{\tau}_{T2}^* = \frac{1}{15}\tau_{\lambda 2}\eta_2(\tilde{z}_2),$$

$$\tilde{\tau}_v = \frac{2}{9}\frac{\rho_2^0 a^2}{\mu_1}, \quad \tilde{\tau}_{\mu 1} = \frac{\rho_1^0 a^2}{\mu_1}, \quad \tilde{\tau}_d = \frac{a^2}{D_1}, \quad \tilde{\tau}_{\lambda j} = \frac{a^2}{\varkappa_j}, \tag{1.5}$$

$$\tilde{y} = \frac{1-i}{\sqrt{2}}(\omega\tilde{\tau}_d)^{1/2}, \quad \tilde{z}_j = \frac{1-i}{\sqrt{2}}(\omega\tilde{\tau}_{\lambda j})^{1/2}, \quad j = 1, 2,$$

$$\varphi(\tilde{y}) = \frac{1}{1+\tilde{y}}, \quad \eta_1(\tilde{z}_1) = \frac{1}{1+\tilde{z}_1}, \quad \eta_2(\tilde{z}_2) = \frac{5\left[3\tilde{z}_2 - (3+\tilde{z}_2^2)\,\mathrm{th}\,\tilde{z}_2\right]}{\tilde{z}_2^2(\mathrm{th}\,\tilde{z}_2 - \tilde{z}_2)}, \quad \varkappa_j = \frac{\lambda_j}{\rho_j^0 c_j}$$

(μ_1 is the dynamic viscosity of the gas, λ is the heat conduction coefficient, D_1 is the binary diffusion coefficient, and β is the accommodation coefficient).

The system of equations (1.1)-(1.5) is closed, and can be used to investigate the propagation of acoustic disturbances in polydisperse mixture of a gas with vapor and liquid drops.

2. We investigate solutions of the system of linear equations (1.1)-(1.5) having the form of progressive waves for the disturbance:

$$\begin{gathered}\psi' = A_\psi \exp i(K_* x - \omega t) = A_\psi \exp(-K_{**}x)\exp[i(Kx - \omega t)], \\ K_* = K + iK_{**}, \quad C_p = \omega/K, \quad C_g = d\omega/dK, \quad \sigma = 2\pi K_{**}/K.\end{gathered} \tag{2.1}$$

Here A_ψ is the complex disturbance amplitude of parameter ψ, i is the imaginary unity, K_* is the complex wave number, K_{**} is the linear attenuation coefficient, and C_p, C_g, and σ are the phase and group velocities, and the attenuation decrement per wavelength. Substituting a solution of the form (2.1) into Eqs. (1.1)-(1.5), we obtain

$$\begin{gathered}
-i\omega A_{\rho i} + i\rho_{i0}K_* A_{v1} + \frac{r\rho_{20}}{p_{10}}\left\langle\frac{A_{pV} - \tilde{A}_{pV\Sigma}}{\tilde{\tau}_p^*}\right\rangle = 0, \quad i = 1, V, \\
-i\omega\tilde{A}_{\rho 2} + iK_*\rho_{20}\langle\tilde{A}_{v2}\rangle - \frac{r\rho_{20}}{p_{10}}\left\langle\frac{\tilde{A}_{pV\Sigma} - \tilde{A}_{pVS}}{\tilde{\tau}_\beta}\right\rangle = 0, \\
-i\omega\rho_{10}A_{v1} + iK_* A_{p1} + \rho_{20}\left\langle\frac{A_{v1} - \tilde{A}_{v2}}{\tilde{\tau}_v^*}\right\rangle = 0, \\
-i\omega\rho_{10}c_{p1}A_{T1} + i\omega\alpha_{10}A_{p1} + r\rho_{20}c_{p1}\left\langle\frac{A_{T1} - \tilde{A}_{T\Sigma}}{\tilde{\tau}_{\Sigma 1}^*}\right\rangle = 0, \\
-i\omega\tilde{A}_{v2} - \frac{A_{v1} - \tilde{A}_{v2}}{\tilde{\tau}_v^*} = 0, \quad -i\omega\tilde{A}_{T2} + \frac{\tilde{A}_{T2} - \tilde{A}_{T\Sigma}}{\tilde{\tau}_{T2}^*} = 0, \\
rc_{p1}\frac{A_{T1} - \tilde{A}_{T\Sigma}}{\tilde{\tau}_{\Sigma 1}^*} + c_2\frac{\tilde{A}_{T2} - \tilde{A}_{T\Sigma}}{\tilde{\tau}_{T2}^*} + \frac{rl_0}{p_{10}}\frac{\tilde{A}_{pV\Sigma} - \tilde{A}_{pVS}}{\tilde{\tau}_\beta} = 0, \\
\frac{A_{pV} - \tilde{A}_{pV\Sigma}}{\tilde{\tau}_p^*} = \frac{\tilde{A}_{pV\Sigma} - \tilde{A}_{pVS}}{\tilde{\tau}_\beta}, \quad \tilde{A}_{pVS} = \frac{l_0\rho_{V0}^0}{(1 - rk_V)T_0}\tilde{A}_{T\Sigma}, \\
A_{pV} - \frac{C_V^2}{\gamma_V\alpha_{10}}(A_{\rho V} + rk_V\tilde{A}_{\rho 2}) - \frac{p_{V0}}{T_0}A_{T1} = 0, \\
A_{p1} - \frac{C_{10}^2}{\gamma_{10}\alpha_{10}}[g_2(A_{\rho 1} + r\tilde{A}_{\rho 2}) + g_1(A_{\rho V} + rk_V\tilde{A}_{\rho 2})] - \frac{p_{10}}{T_0}A_{T1} = 0 \\
\left(\tilde{\tau}_{\Sigma 1}^* = \frac{\alpha_{20}}{\alpha_{10}}\tilde{\tau}_{T1}^*\right).
\end{gathered} \tag{2.2}$$

Here the linear averaging operator is [13]

$$\langle\tilde{h}\rangle = \left(\int_{a_{\min}}^{a_{\max}}\tilde{N}_0\tilde{h}a^3\,da\right)\Bigg/\left(\int_{a_{\min}}^{a_{\max}}\tilde{N}_0 a^3\,da\right) = \frac{1}{\rho_{20}}\int_{a_{\min}}^{a_{\max}}\tilde{N}_0\tilde{m}_{20}\tilde{h}\,da.$$

According to the definition we have for the gas phase parameter $A_{\psi 1}$, due to its independence of the parameter a, $\langle A_{\psi 1}\rangle \equiv A_{\psi 1}$.

To bring out the independent amplitudes $\tilde{A}_{pV\Sigma}$, $\tilde{A}_{pVS}$, $\tilde{A}_{T\Sigma}$, $\tilde{A}_{v2}$ from under the averaging operator < > in Eqs. (2.2), we express them in terms of the amplitudes of the gas phase parameters $A_{\psi 1}$, which are independent of a, and the known characteristic times $\tilde{\tau}^*$:

$$\begin{gathered}\tilde{A}_{v2}=(1-i\omega\tilde{\tau}_v^*)^{-1}A_{v1},\quad \tilde{A}_{pV\Sigma}=(\tilde{\tau}_p^*+\tilde{\tau}_\beta)^{-1}(\tilde{\tau}_\beta A_{pV}-\tilde{\tau}_p^*\tilde{A}_{pVS}),\\ \tilde{A}_{pVS}=\frac{l_0\rho_{V0}^0}{(1-rk_V)T_0}\tilde{A}_{T\Sigma},\quad \tilde{A}_{T\Sigma}=\tilde{Z}\left[A_{T1}+i\omega\tilde{\tau}_{\Sigma1}^*\tilde{e}_2\left(\frac{l_0}{p_0c_{p1}}\right)A_{pV}\right],\\ \tilde{Z}=\left[1-i\omega\tilde{\tau}_{\Sigma1}^*\left(\tilde{e}_1-\frac{\rho_{V0}^0}{p_{10}}l_0(\gamma_1-1)\tilde{e}_2\right)\right]^{-1},\\ \tilde{e}_1=\frac{c_2}{rc_{p1}}(1-i\omega\tilde{\tau}_{T2}^*)^{-1},\quad \tilde{e}_2=[i\omega(\tilde{\tau}_p^*+\tilde{\tau}_\beta)]^{-1}.\end{gathered}\tag{2.3}$$

Substituting expressions (2.3) into system (2.2), we obtain a homogeneous system of linear algebraic equations in the amplitudes A_ψ.

The determinant of coefficients of the unknown A_ψ must vanish, so that a nonvanishing solution of this system exist. Decreasing the order of the determinant, following some algebraic transformations we have the following dispersion relation for the wave number

$$(C_1K_*/\omega)^2=V(\omega)\,D(\omega),\tag{2.4}$$

where V(ω), D(ω) are complex functions, describing the effects of sound dispersion and dissipation in the suspension due to the processes of interphase friction and interphase heat and mass exchange, respectively. In the absence of particles (m = 0) V(ω) = D(ω) = 1, i.e., there are no dispersion and dissipation in a gas without particles. The functions V(ω), D(ω) depend on frequency, on the thermophysical phase parameters, and on the spectral composition of the mixture in terms of the functions written below:

$$\begin{gathered}V(\omega)=1+mV^0(\omega),\quad V^0(\omega)=\langle\tilde{h}_v\rangle,\\ \tilde{h}_v=(1-i\omega\tilde{\tau}_v^*)^{-1},\quad \tilde{\tau}_v^*=\tilde{\tau}_v\left[1+\frac{1-i}{\sqrt{2}}(\omega\tilde{\tau}_{\mu1})^{1/2}\right]^{-1},\\ D(\omega)=1+mr(\gamma_1-1)\frac{H_2-bk_V\gamma_1(b\bar{c}_1H_3-2\bar{l}H_1)-M_1\Lambda}{1+mr(H_2-BH_3-M_2\Lambda)},\\ H_j=\langle\tilde{h}_j\rangle\quad(j=1-3),\quad \Lambda=LH_1^2+H_2H_3,\\ \tilde{h}_1=\tilde{Z}\tilde{e}_2,\quad \tilde{h}_2=\tilde{Z}(\tilde{e}_1-L\tilde{e}_2),\quad \tilde{h}_3=\tilde{Z}[\tilde{e}_2(1-i\omega\tilde{\tau}_{\Sigma1}^*\tilde{e}_1)],\\ \tilde{Z}=[1-i\omega\tilde{\tau}_{\Sigma1}^*(\tilde{e}_1-L\tilde{e}_2)]^{-1},\quad \tilde{e}_1=\frac{\bar{c}_2}{r\bar{c}_1}(1-i\omega\tilde{\tau}_{T2}^*)^{-1},\quad \tilde{e}_2=[i\omega(\tilde{\tau}_p^*+\tilde{\tau}_\beta)]^{-1},\\ M_1=mrb\bar{c}_1(\gamma_1-1+bk_V),\ M_2=mrB,\ L=\gamma_1(\gamma_1-1)k_V\bar{l}^2,\\ \bar{l}=\frac{l_0}{C_{10}^2},\quad b=\frac{R_V}{R_{10}},\quad B=(1-k_Vb)\,b,\quad r=\frac{\rho_{10}^0}{\rho_{20}^0},\\ \bar{c}_1=\frac{c_{p1}}{\gamma_1R_1}=\frac{1}{\gamma_1-1},\quad \bar{c}_2=\frac{c_2}{\gamma_1R_1}.\end{gathered}\tag{2.5}$$

We note that the dispersion dependence of (2.4), (2.5) has been obtained for the case of low bulk content ($\alpha_2 \ll 1$) and moderate pressures ($r \ll 1$). In this case, however, the drop mass content can be quite high ($m \sim 1$). Account of terms with α_2 and r leads to the appearance of factors of the form $(1 - \alpha_2)$ and $(1 - r)$ in the dispersion relation.

In the special cases of single-component mixtures of vapor with drops ($k_V = 1$) and a gas with particles, in the absence of phase transformations ($k_V = 0$, $\tilde{\tau}_\beta = \infty$) the dispersion dependence (2.4), (2.5) is in agreement with the dependencies of [13]. The dispersion relation for a monodisperse vapor–gas–drop mixture [8] is obtained from (2.4), (2.5) with the substitution $N_0(a) = n_0\delta(a - a_0)$, where δ denotes the Dirac δ-function, while $\langle\tilde{h}_j\rangle = h_j(a_0, \omega)$ (j = 1-3).

3. Consider the special case of low drop mass content ($m \ll 1$), when the effect of interphase mass exchange on acoustic wave propagation is most substantial [8-11]. Neglecting in (2.4), (2.5) terms of order higher than m, the order of smallness, a simpler relation is

obtained, describing the dispersion and dissipation effects in vapor–gas–drop media of the aerosol fog type:

$$C_1K_*/\omega = 1 + \frac{m}{2}[V^0(\omega) + rD^0(\omega)],$$
$$D^0(\omega) = (\gamma_1 - 1)\frac{H_2 - bk_V\gamma_1(b\bar{c}_1H_3 - 2\bar{l}H_1)}{1 + mr(H_2 - BH_3)}. \quad (3.1)$$

Here the functions V^0, H_j (j = 1-3) have the same form as in (2.5).

It must be stressed that according to (3.1) the contributions of the friction phases and of interphase heat and mass exchange to the disturbance dispersion and dissipation in the aerosols are additive. Unlike a gas with solid particles [13], however, the contribution of interphase heat and mass exchange, determined by the function $D^0(\omega)$, is not proportional to the fractional masses of different size particles. Consequently, relation (3.1) cannot be obtained by simple integration of the corresponding dependencies for a monodisperse suspension in fractional masses, as in the case of a gas suspension without phase transformations. In this connection we also note that a similar dispersion dependence for a vapor–drop aerosol [13], obtained by formal transition to the case of small m from the general dependence, does not describe a number of effects of interphase mass exchange, particularly the nonmonotonic character of sound damping in aerosols [14], the limiting transitions for $\omega \to 0$ and $\beta \to \infty$, and so on. The corresponding dependence for a vapor and drop mixture can be obtained from relation (3.1) for $k_V = 1$.

We investigate the low-frequency asymptote of the complex wave number $K_* = K + iK_{**}$, following from the dispersion relation (3.1) for $\omega \to 0$. To simplify the analysis we neglect the nonequilibrium interphase surfaces with mass exchange, and assume that $R_V \sim R_G$ and $\varkappa_1 \sim D_1$.

We further use the dimensionless parameters characterizing the thermophysical and acoustic properties of aerosol:

$$\Pr_1 = \mu_1 c_{p1}/\lambda_1, \quad \bar{K}_* = K_* C_1 \tau_{v*}, \quad \Omega = \omega\tau_{v*},$$
$$\bar{C}_p = C_p/C_1, \quad \bar{a} = a/a_* \qquad (\tau_{v*} = (2/9)\rho_2^0 a_*^2/\mu_1)$$

(a_* is a representative radius).

The corresponding low-frequency asymptote $\bar{K}^*$ can be written in the form

$$\left(\frac{\bar{K}_*}{\Omega}\right)_{\Omega\to 0} \sim (\bar{C}_e)^{-1} + \frac{i}{2}\left\{m\Lambda_1\bar{a}_{5,3}^2 + \frac{k_V}{m}\Lambda_2\bar{a}_{3,1}^2\right\}\Omega,$$
$$\Lambda_1 = 1 + \frac{3}{2}\Pr_1(\gamma_1 - 1)\left[(1 - k_V)\frac{c_2}{c_1}\right]^2, \quad (3.2)$$
$$\Lambda_2 = \frac{3}{2}\Pr_1(\gamma_1 - 1)(\bar{l} - \bar{c}_1)^2\gamma_1/\Lambda_3, \quad \Lambda_3 = \gamma_1\bar{l}^2k_V + (1 - k_V)\bar{c}_1,$$

where $a_{i,j}$ are mean radii, determined by the equation

$$a_{i,j} = \left\{\left[\int_{a_{\min}}^{a_{\max}} N_0(a)a^i\,da\right]\Big/\left[\int_{a_{\min}}^{a_{\max}} N_0(a)a^j\,da\right]\right\}^{1/(i-j)}, \quad i \neq j,$$
$$a_{\min} \leqslant a_{i,j} \leqslant a_{\max}; \quad (3.3)$$

and $\bar{C}_e$ is the dimensionless equilibrium sound velocity in a gas mixture with a vapor and liquid drops [8]. At low drop mass content ($m \ll 1$) we write down for $(\bar{C}_e)^{-1}$

$$(\bar{C}_e)^{-1} = 1 + \frac{1}{2}\left\{m\left[1 + (1 - k_V)(\gamma_1 - 1)\frac{c_2}{c_1}\right] + \gamma_1 k_V(\gamma_1 - 1)(\bar{l} - \bar{c}_1)^2/\Lambda_3\right\}.$$

We note that in the asymptote (3.2) the relation with the coefficient Λ_2 is related to effects of interphase mass exchange. In this cae, due to the smallness of m ($\ll 1$) for vapor concentrations $k_V > m$ the term with Λ_2 is dominant in the factor 1/m. Thus, the attenuation of low-frequency disturbances in polydisperse vapor–gas–drop systems in a wide range of variation of vapor concentration ($m < k_V \leqslant 1$) is basically determined by phase transformation effects. It must be stressed, however, that for low disturbance frequencies, with decreasing drop mass content the m linear solutions may become inadequate [14]. For ordinary aerosols in the presence of phase transformations the linear analysis is effective when $m \gtrsim 10^{-3}$ [14].

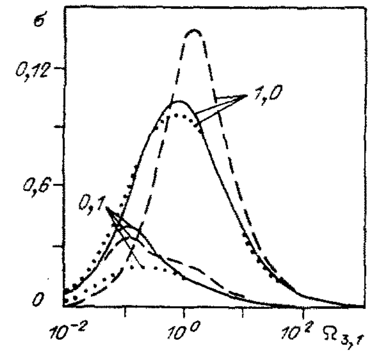

Fig. 1

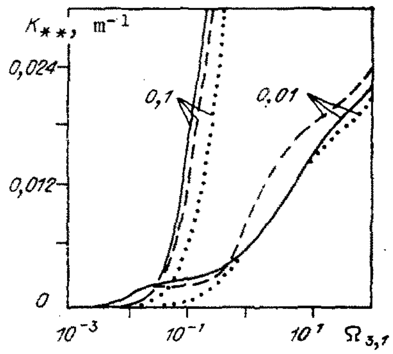

Fig. 2

According to (3.2), account of the effect of spectral composition on the propagation of low-frequency disturbances in polydisperse vapor–gas–drop mixtures with an arbitrary shape of the distribution function $N_0(a)$ reduces only to account of the integral characteristics $a_{3,1}$ and $a_{5,3}$. In this case the mean radius $a_{5,3}$ is related to interphase friction effects, and for $k_V \neq 1$ – to interphase heat exchange effects, while the radius $a_{3,1}$ is basically related to phase transformation effects. This fact must be taken into account in analyzing the characteristic times [10] of interphase interaction effects of polydisperse suspensions with mass exchange. Since in the general case $a_{3,1} \neq a_{5,3}$, the propagation of low-frequency sound in systems of the type of polydisperse fog cannot be described within monodisperse models, unlike the case of suspensions without mass exchange.

4. Several results on dissipation and dispersion of monochromatic waves in mixtures of air with vapor and water drops with a gas phase pressure $p_1 = 0.1$ MPa and vapor concentration $k_V = 0.1$ ($T_0 = 327$ K), obtained by means of the dispersion relations (2.4), (2.5), are illustrated in Figs. 1-4.

The dispersion curves are shown in the form of attenuation coefficients σ, K_{**} and the phase velocity C_p as a function of the dimensionless frequency $\Omega_{3,1}$ ($\Omega_{3,1} = \omega\tau_{V*}$, $a_* = a_{3,1}$). As a dimension-removing parameter we use the radius $a_{3,1}$, which according to the asymptotic K_* (3.2) is the characteristic mean radius of the polydisperse vapor–gas–drop mixture during low-frequency acoustic actions, which is usually quite close in value to that of $a_{3,2}$, is also characteristic of propagation of high-frequency disturbances in polydisperse suspensions without mass exchange [13]. In this case, according to the Hölder inequality, the $a_{3,1}$ is the minimum among the other characteristic mean sizes of the polydisperse suspension ($a_{3,1} \leqslant a_{3,2} \leqslant a_{5,3}$). Therefore, the use of a_* as dimension-removing parameter rather than others, such as $a_{5,3}$, leads to a shift in the dispersion curves along the abscissa axis $\eta = \log\Omega$ toward increasing η, since $\log\Omega_{5,3} = \log\Omega_{3,1} + 2\log(a_{5,3}/a_{3,1})$ ($a_{5,3}/a_{3,1} \geqslant 1$).

Further, to illustrate sound propagation in aerial fog we selected a uniform mass distribution of drops, when $N(a) = \text{const}\, a^{-3}$, $a_{min} = 3$ μ, $a_{max} = 30$ μ, $a_{3,1} = 40$ μ. The constant value in the function $N(a)$ can be determined from the values of the bulk drop content in the mixture α_2 or the mass concentration m. According to (2.5), (3.3), however, the value of this constant does not affect the shape of the dispersion curves and the values of the mean radii $a_{i,j}$.

Figures 1-3 show the attenuation decrement per wave length σ, the linear attenuation coefficient K_{**}, and the phase velocity C_p as a function of the frequency $\Omega_{3,1}$ for various drop mass contents m. The solid and dotted curves correspond to the cases of nonequilibrium ($\beta = 0.04$) and frozen ($\beta = 0$) mass exchange between the drops and the gas in the polydisperse fog. The dashed lines are used to illustrate the dispersion curves, corresponding to monodisperse vapor–gas–drop mixtures with a drop radius $a_0 = a_{3,1} = 10$ μ in the presence of a nonequilibrium phase transformation. The digits on the curves indicate the value of the mass content of the suspended phase m.

Analysis shows that for very small m (≤ 0.01) the shape of the dependence $\sigma(\Omega_{3,1})$ of a polydisperse fog is basically determined by mass exchange effects, and practically coincides with the corresponding dependence for a monodisperse mixture [8]. With increasing m the contribution of interphase friction to the disturbance dissipation increases, and for m = 0.1 the solid and dashed curves differ (Fig. 1). This difference is most substantial at intermediate frequencies ($\Omega_{3,1} \sim 1$), when the effect of a polydisperse composition of a suspension on sound propagation is maximum and cannot be described within a monodisperse model with the use of effective sizes (Figs. 1-3). Therefore, neither of the mean

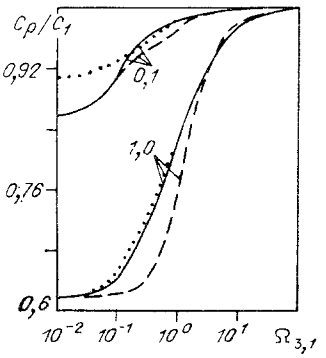

Fig. 3

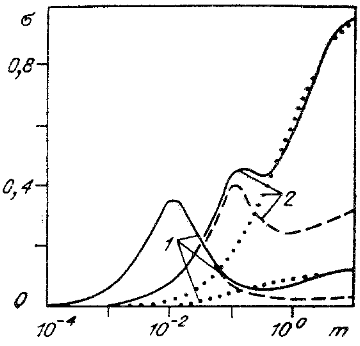

Fig. 4

radii (3.3) or [16] can be used as a characteristic size in the given frequency region. We note that for sufficiently high drop mass content ($m \sim 1$) the attenuation decrement σ for a monodisperse aerosol can both substantially exceed and be substantially less than σ for a polydisperse suspension at different frequencies $\Omega_{3,1}$. The effect of interphase mass exchange on wave propagation in polydisperse fogs, as well as for monodisperse suspensions [8], is most sharply expressed in aerosols with small m (the solid and dotted curves of Figs. 1-3). The role of multirate effects increases and the difference between these curves decreases with increasing m.

The substantial effect of phase transformations on the propagation of low-frequency disturbances in polydisperse aerosols with small m leads, as is the case for monodisperse suspensions [14], to the anomalous effect of nonmonotonic dependence of sound dissipation on the drop mass content m. This effect is unexpected, since according to the commonly accepted point of view the intensity of disturbance attenuation in these systems is proportional to the mass content of the disperse phase, which is the source and the fundamental reason for dissipation.

Figure 4 illustrates the m-dependence of σ for fixed dimensionless frequencies: 1) $\Omega_{3,1} = 0.01$ ($\omega = 10\ \sec^{-1}$), 2) $\Omega_{3,1} = 0.1$ ($\omega = 100\ \sec^{-1}$). It is seen that the dependence $\sigma(m)$ is nonmonotonic, and has a local maximum when $m \sim \Omega_{3,1}$. For a monodisperse fog, however, the maximum of σ at frequency $\Omega_{3,1} = 0.1$ is more sharply expressed (dashed curve). It is noted that for $m \ll 1$ the intensity of attenuation in a suspension with phase transformations substantially exceeds the attenuation in a gas with particles, where these transformations are absent (dotted curves). With increasing m the effect of interphase mass exchange on disturbance attenuation with the frequencies considered drops (the solid and dotted curves practically coincide for $m \gg 1$). In this case the disturbance attenuation in polydisperse gas suspensions with high drop mass content ($m \sim 10$) can substantially exceed (by 3 times) the wave dissipation in monodisperse systems (the solid and dotted curves 2).

The authors are grateful to S. A. Yakubov for performing the numerical calculations.

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EFFECT OF THE MOTION INSIDE A LIQUID DROP ON ITS RISE IN A VERTICAL TUBE

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UDC 532.529.6

1. Introduction. In treating the rise of bubbles in a liquid it is usually assumed that the medium inside the bubble is at rest and the state of the medium can be described by a single constant: the thermodynamic pressure p_g of the gas inside the bubble. For gas or air bubbles rising in a heavy liquid this assumption is justified, since the ratios of the densities and viscosities of the gas and liquid are small and so the medium inside the bubble is light and the friction of the gas against the liquid on the surface of the bubble is small and has a negligible effect on the motion. This assumption is supported by numerous experiments. However for vapor bubbles, such as in Freon, the ratio of the densities is of order 0.1 and the use of the bubble model can lead to inaccurate results.

2. Statement of the Problem and Solution Algorithm. We assume that the medium inside the bubble is a viscous incompressible liquid (a liquid drop moving in a different liquid) with $\rho_1/\rho_2 = 0.1$. We consider hindered motion of the liquid drop in a tube with $\lambda = 0.8$. Here ρ_1 and ρ_2 are the densities of the media inside and outside the liquid drop; $\lambda = a/R_k$, where a is the radius of a sphere whose volume is equal to that of the liquid drop and R_k is the radius of the tube. Since the liquid drop occupies more than half of the tube cross section, the flow of the liquids inside and outside the drop are determined by the nature of the flow through the narrow gap between the wall of the tube and the surface of the drop. For small λ the effect of the tube wall is small, as is shown by calculations of rising bubbles [1], but its effect increases with λ.

The motion inside and outside the liquid drop is described by the Navier–Stokes equations. Consistency conditions must be satisfied on the interface Γ between the two liquids [2]. The velocities and the tangential components of the stress must be equal across the interface, while the normal component of the stress has a jump equal to the magnitude of the capillary pressure. The algorithm for obtaining the numerical solution of the problem is constructed in analogy to [1] and has been described in detail in [3]. The results of a series of calculations are summarized in Fig. 1 in terms of the coordinates $R_\sigma = a/(\nu_2^2/g)^{1/3}$, $R_\nu = a/(\sigma/\rho_2 g)^{1/2}$. A given external medium corresponds to a straight line on the diagram, since $R_\sigma/R_\nu = (g\rho_2^3\nu/_2^4\sigma^3)^{1/6} = M^{1/6}$, where M depends only on the physical constants of the external medium (g is the acceleration of gravity, σ is the surface tension on the interface between the media, and ν_1 and ν_2 are the kinematic viscosities of the liquid drop and the external liquid). Because of the large number of dimensionless parameters [3] (we use the Reynolds numbers of the internal and external fluids $Re_1 = u2a/\nu_1$, $Re_2 = u2a/\nu_2$ and the Weber number $We = \rho_2 u^2 2a/\sigma$) it is difficult to generalize the results. Nevertheless the lines of

Novosibirsk. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, No. 4, pp. 83-94, July-August, 1993. Original article submitted August 13, 1992.

 0021-8944/93/3404-0524$12.50